



ENVIRONMENTAL PROTECTION AGENCY

APTI 413: Control of Particulate Matter Emissions

Student Manual:

Chapter 3

APTI: 413 CONTROL OF PARTICULATE MATTER EMISSIONS, 5TH EDITION

Student Manual



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The preparation of this manual was overseen by the organizations of the Environmental Protection Agency (EPA) and the National Association of Clean Air Agencies (NACAA*), and the revision of materials was coordinated and managed by the Tidewater Operations Center of C² Technologies, Inc., Newport News, Virginia.

Valuable research and feedback was provided by an advisory group of subject matter experts composed of Dr. Jerry W. Crowder, TX; Dr. Tim Keener, OH; Dr. Douglas P. Harrison, LA., and Mr. Tim Smith, Senior Air Quality Specialist, EPA, Office of Air Quality Planning and Standards.

Acknowledgements and thanks are also extended to Rutgers University, especially Francesco Maimone, Co-Director of the Rutgers Air Pollution Training Program, which hosted the pilot of the revised materials, and the additional reference sources that can be found throughout the manuals.

**The National Association of Clean Air Agencies (NACAA) represents air pollution control agencies in 53 states and territories and over 165 major metropolitan areas across the United States.*

State and local air pollution control officials formed NACAA (formerly STAPPA/ALAPCO) over 30 years ago to improve their effectiveness as managers of air quality programs. The associations serve to encourage the exchange of information among air pollution control officials, to enhance communication and cooperation among federal, state, and local regulatory agencies, and to promote good management of our air resources.

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Chapter 3

This chapter will take approximately 1.75 hours to complete.

Particle Sizing

This chapter explains the range of different particle sizes, tools used to determine particle sizes and the calculations that permiters will use to determine particle size from industrial sources.

OBJECTIVES

Terminal Learning Objective

At the end of this chapter, the student will be able to understand methods available for determining particle size distributions from emission sources and mathematical methods for representing those data.

Enabling Learning Objectives

- 3.1 Determine the range of particle sizes of concern in air pollution control .
- 3.2 Recognize the techniques and limitations of several particle sizing methods.
- 3.3 Determine mathematical representations of log-normal particle size distribution.

Checks on Learning

Problem Examples
And
End of Chapter
Review Problems

3.1 Particle Size

The range of particle sizes of concern in air pollution control is extremely broad. Some of the droplets collected in the mist eliminators of wet scrubbers and the solid particles collected in large diameter cyclones are as large as raindrops. Some of the small particles created in high temperature incinerators and metallurgical processes are so small that more than 500 of them could be lined up across the diameter of a human hair.

To appreciate the difference in sizes, it is helpful to compare the diameters, areas, and volumes of a variety of particles. Assume that all of the particles are simple spheres. The raindrop shown in Figure 3-1 is 500 μm in diameter. The term micrometer (μm) simply means one millionth of a meter. One thousand micrometers are equivalent to 0.1 cm or 1.0 μm . In some texts, the term micron is often used as an abbreviation for micrometer.

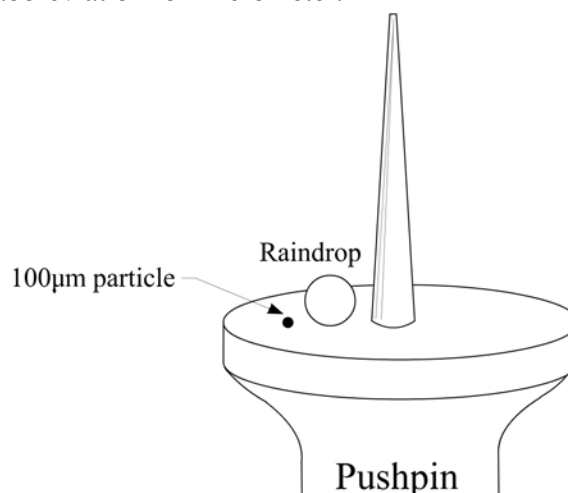


Figure 3-1. Very large particle and a raindrop

A 100 μm particle shown next to the raindrop in Figure 3-1 looks like a small speck compared to the pushpin. However, both the raindrop and the 100 μm particle are on the large end of the particle size range of interest in air pollution control. Particles in the range of 10-100 μm are also on the large end of the particle size scale of interest in this course.

The particle size range between 1 and 10 μm is especially important in air pollution control. A major fraction of the particulate matter generated in some industrial sources is in this size range. Furthermore, all particles less than or equal to 10 μm are considered respirable and are regulated as PM_{10} . Figure 3-2 shows a comparison of 1, 10, and 100 μm particles. Note the substantial difference in size between these particles.

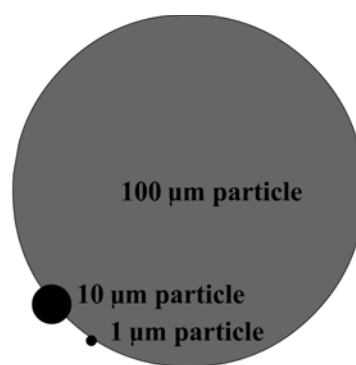


Figure 3-2. 1 μm and 10 μm particles compared to a 100 μm particle.

Particles in the range of 0.1 to 1.0 μm are important in air pollution control because they can represent a significant fraction of the particulate matter emissions from some types of industrial sources and because they are relatively hard to collect. Some industrial processes, such as combustion and metallurgical sources, generate particles in the range of 0.01 to 0.1 μm . These sizes are approaching the size of individual gas molecules, which are in the range of 0.0002 to 0.001 μm . However, particles in this size range tend to agglomerate rapidly to yield particles in the greater than 0.1 μm range. Accordingly, very little of the particulate matter entering an air pollution control device remains in the size range of 0.01 to 0.1 μm .

Throughout this manual, small particles are defined as less than 1 μm , moderately-sized particles are classified as 1 to 10 μm , and large particles are classified as 10 to 1,000 μm . The volumes and surface areas of particles over this size range are shown in Table 3-1.

The data in Table 3-1 indicate that 1,000 μm particles are more than 1,000,000,000,000 times (one trillion) larger in volume than 0.1 μm particles. As an analogy, assume that a 1,000 μm particle was a large domed sports stadium. A basketball in this stadium would be equivalent to a 5 μm particle. Approximately 100,000 spherical particles of 0.1 μm diameter would fit into this 5 μm basketball.

The entire 1,000 μm stadium is the size of a small raindrop.

Particles over this extremely large size range of 0.1 to 1,000 μm are of interest in air pollution control.



**Change in D of 1x
= change in volume
of x^3**

Table 3-1. Spherical Particle Diameter, Volume, and Surface Area		
Diameter (μm)	Volume (cm^3)	Area (cm^2)
0.1	5.23×10^{-16}	3.14×10^{-10}
1.0	5.23×10^{-13}	3.14×10^{-8}
10.0	5.23×10^{-10}	3.14×10^{-6}
100.0	5.23×10^{-7}	3.14×10^{-4}
1,000.0	5.23×10^{-4}	3.14×10^{-2}

Particle size itself is difficult to define in terms that accurately represent the types of particles. This difficulty stems from the fact that particles exist in a wide variety of shapes, not just as spheres as shown earlier. The photomicrograph shown in Figure 3-3 has a variety of spherical particles and irregularly shaped particles. For spherical particles, the definition of particle size is simply the diameter. For the irregularly shaped particles, size can be defined in a variety of ways. For example, when measuring the size of particles on a microscope slide, size can be based on the diameter of the particle parallel to the microscope scan that divides the particle into equal areas (Martin's diameter) or the mean length between two tangents on opposite sides of the particle that are perpendicular to the fixed direction of the microscope scan (Ferret's diameter).



**Particle size is
difficult to
determine because
particles are not all
spheres.**

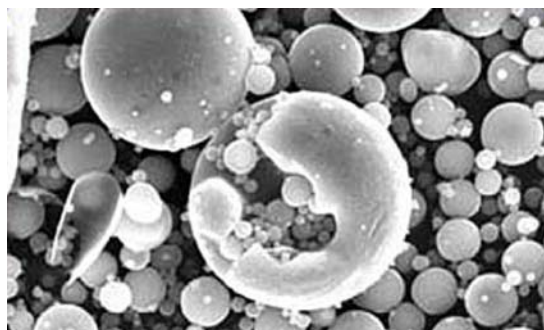


Figure 3-3. Photomicrograph of coal fly ash

Neither of these microscopically based size definitions, however, is directly related to how particles behave in a fluid such as air. The particle size definition that is most useful for evaluating particle motion in a fluid is termed the aerodynamic diameter. For all particles greater than 0.5 μm , the aerodynamic diameter can be approximated by:

(3-1)




$$d_p = d\sqrt{\rho_p C_c}$$

Where




 d_p = aerodynamic particle diameter (μm) d = physical diameter (μm) ρ_p = particle density (g/cm^3) C_c = Cunningham slip correction

The Cunningham slip correction accounts for particle motion through a non-continuous medium and become increasingly more significant as the particle diameter decreases below about 3 μm . Calculation of the Cunningham slip correction factor will be discussed in Chapter 4.

Aerodynamic particle diameter is determined by inertial sampling devices such as the cascade impactor, which is discussed later in this chapter. Particles that appear to be different in physical size and shape can have the same aerodynamic diameter, as illustrated in Table 3-2. Conversely, as illustrated in Table 3-3, some particles that appear to be visually similar can have somewhat different aerodynamic diameters.

Table 3-2. Aerodynamic Diameters of Differently Shaped Particles			
	Solid Sphere	$\rho_p = 2.0 \text{ g}/\text{cm}^3$ $d = 1.4 \mu\text{m}$	$d_p = 2.0 \mu\text{m}$
	Hollow Sphere	$\rho_p = 0.50 \text{ g}/\text{cm}^3$ $d = 2.80 \mu\text{m}$	
	Irregular Shape	$\rho_p = 2.3 \text{ g}/\text{cm}^3$ $d = 1.3 \mu\text{m}$	

The term *aerodynamic diameter* is useful for all particles including the fibers and particle clusters shown in Figure 3-4. The aerodynamic diameter provides a simple means of categorizing the sizes of particles with a single dimension and in a way that relates to how particles move in a fluid. Unless otherwise noted, particle size is expressed in terms of aerodynamic diameter throughout the remainder of this manual.

Table 3-2. Aerodynamic Diameters of Particles With Different Densities		
	$\rho = 1.0 \text{ g/cm}^3$ $d = 2.0 \text{ }\mu\text{m}$	$d_p = 2.0 \text{ }\mu\text{m}$
	$\rho = 2.0 \text{ g/cm}^3$ $d = 2.0 \text{ }\mu\text{m}$	$d_p = 2.8 \text{ }\mu\text{m}$
	$\rho = 3.0 \text{ g/cm}^3$ $d = 2.0 \text{ }\mu\text{m}$	$d_p = 3.5 \text{ }\mu\text{m}$

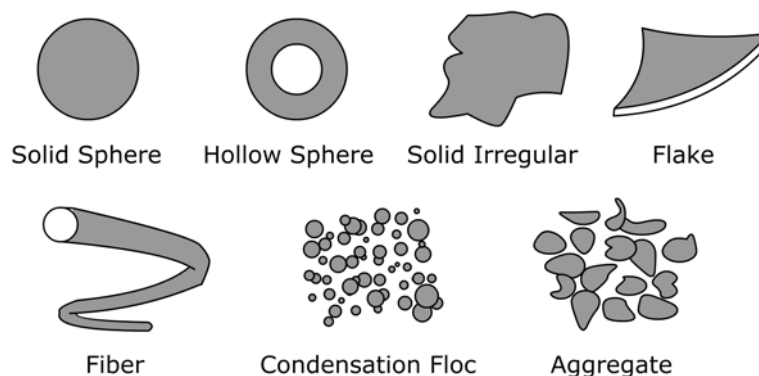


Figure 3-4. Different shapes of particles

3.2 Particle Size Measurement

Several alternative methods are used to evaluate the size distribution of particulate matter in industrial gas streams. An ideal particle-measuring device would be able to do the following:

- Measure the exact size of each particle
- Determine the composition of each particle
- Report real-time data instantaneously

It would be an extremely difficult task to produce such an instrument. At this time, there are devices that incorporate only one or two of these ideal functions. In this section, various sizing techniques will be examined and compared to such an ideal device, listing advantages and disadvantages of each. While this discussion is not intended to be exhaustive, it will review the more commonly employed methods.

Microscopy

Various types of microscopy analyses can be performed on filters or slides that have been exposed to the gas stream. The representativeness of the sample depends in part on the characteristics of the sampling equipment and in part on the adequacy of the sampling procedures. It is important to minimize the sampling times to avoid overloading the surface. Particles should be deposited as a single layer to the extent possible. Sampling times are usually in the range of one to five minutes.

There are several common types of microscopic analyses used to evaluate particle size. Polarizing light microscopy (PLM) uses visible light that is focused on the particle and magnified in a set of lenses mounted in a conventional microscope (Figure 3-5). With the appropriate lenses and sample preparation techniques, PLM analyses can be used to size particles as small as 3 micrometers. Generally speaking, the chemical composition of the particle cannot be determined by using an optical microscope. However, a subsequent chemical analysis can be performed on the sample.



Figure 3-5. Optical microscope used for PLM analyses



**Microscopy is
extremely time
consuming**

The size of a particle is estimated by comparing each particle to a scale in the eyepiece, usually calibrated to micrometers. Each particle, presented in a fixed area of the eyepiece, is sized and tallied into a number of size categories. The number of particles sized may range from 100 to several thousand per sample, depending on the accuracy desired. Typically, 400 or more particles per filter sample section are individually counted to compile a particle size distribution estimate. This method can be time consuming and extremely tedious. Training is needed to properly identify and size particles using this technique.

The particles are usually collected by deposition on a glass slide or filter and later analyzed by a microscope in the lab. In general, size distributions determined from particles collected in the field and transported to the lab must be viewed with caution. First of all, it is difficult to collect a truly representative sample, and then it is almost impossible to maintain the original size distribution while transferring the samples to the laboratory. For example, it is not known whether agglomerate seen on the sample were agglomerates in the gas stream or existed as separate particles. As a result, methods that involve analysis of bulk samples generally overestimate

particle size. In spite of the limitations of the microscopic method, this method is useful in the determination of some properties of interest.

Scanning electron microscopy (SEM) can provide greater magnification of the particles than with PLM. With SEM, it is possible to resolve particles as small as 0.3 micrometers with normal electron beam energy levels. Furthermore, an electron beam microprobe can be used to obtain elemental chemical analyses of individual small particles and even localized areas of large particles. These electron beam analyses are usually termed *energy dispersive x-ray spectroscopy* or EDX. As in the case with PLM, particle size distributions determined by SEM involve the comparison of the projected area of the particle with a calibrated graticule mounted into the viewing port of the SEM.



**Optical counters
are only as good as
the instrument
calibration**

**But can determine
particles as small
as 0.3 μm**

Optical Counters

Optical particle counters have not been widely used for particle sizing because they cannot be directly applied to the stack exhaust gas stream. The sample must be extracted, cooled, and diluted before entering the counter. This procedure must be done with extreme care to avoid introducing serious errors in the analyses. A major benefit of an optical counter is the ability to observe emission (particle) fluctuations on a real time, instantaneous basis. Particles as small as 0.3 μm can be determined with an optical counter.

Optical particle counters work on the principle of light scattering. Each particle in a continuously flowing sample stream passes through a small illuminated viewing chamber. Light scattered by the particle is observed by the photo detector during the time the particle is in the viewing chamber (Figure 3-6). The intensity of the scattered light is a function of particle size, shape, and index of refraction. Optical counters give reliable particle size information only when one particle is in the viewing chamber at a time. The simultaneous presence of more than one particle can be interpreted by the photo detector as a larger sized particle. This error can be minimized by maintaining sample dilution ratios to ensure less than 300 particles per cubic centimeter.

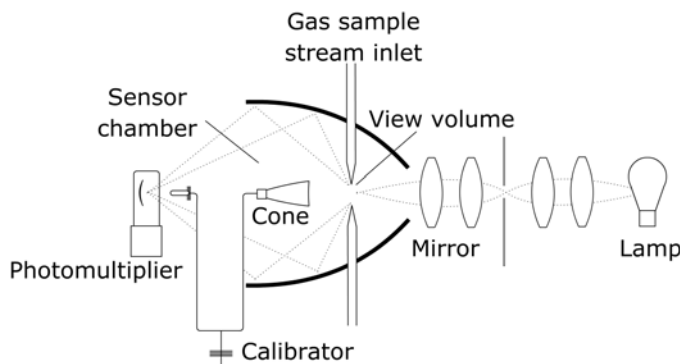


Figure 3-6. Optical particle counter



Bachco analyzers use centrifugal force to separate particles from 1 to 60 μm

Provides information on aerodynamic particle size.



EAA takes advantage of a particle's mobility to determine its size.

As mobility increases, particle size decreases.

A drawback of the optical counter is the dependence of the instrument calibration on the index of refraction and the shape of the particle. Errors in counting can also occur due to the presence of high concentrations of very small particles, which are sensitive to the light wavelength used.

Bachco Analyzer

The Bachco analyzer (Figure 3-7) uses centrifugal force to separate particles ranging in size from 1 to 60 μm . A weighed sample, usually 5 grams, is introduced into a rotating gas stream. The larger particles move to the wall of the chamber and are separated. The rotational speed is then increased in steps in order to separate smaller and smaller size fractions. The separated size fractions are weighed to determine the mass in each size range. Chemical analysis could also be done on each size fraction.

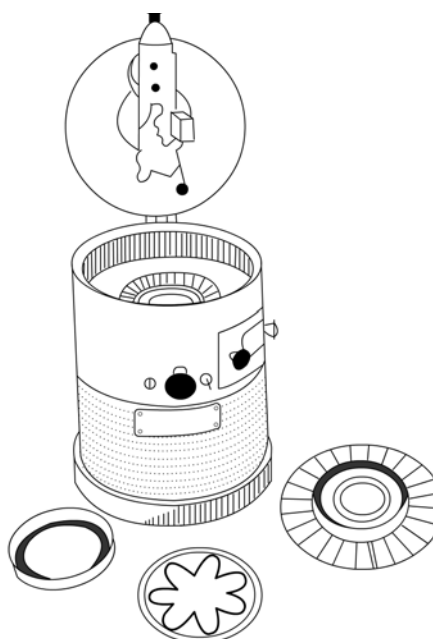


Figure 3-7. Bachco analyzer

The Bachco provides information on aerodynamic particle size. However, because the analysis is performed on a bulk sample, it suffers from the same agglomeration related problems discussed for microscopy and, likewise, tends to overestimate particle size.

Electrical Aerosol Analyzer

The electrical aerosol analyzer (EAA) is a sub micrometer particle size-measuring device commercially developed at the University of Minnesota. The EAA uses an electrical field to separate particles ranging in size from 0.003 to 0.5 μm on the basis of their mobility, a diffusional property that increases as the particle size decreases. One type of EAA is shown in Figure 3-8.

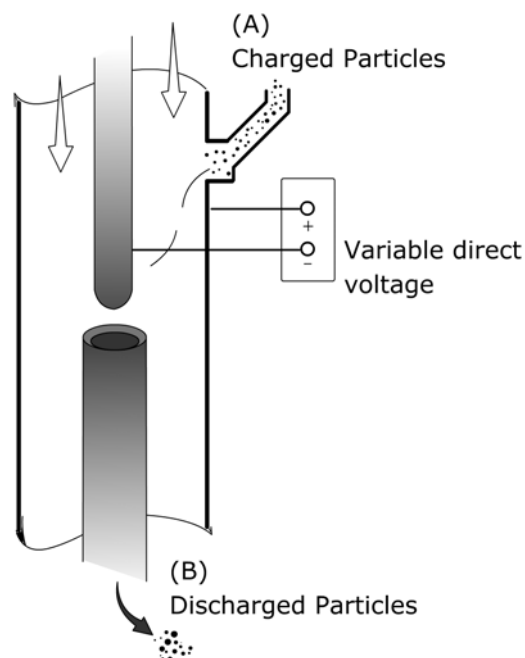


Figure 3-8. Electrical aerosol analyzer

A particle-laden sample stream is extracted from the stack and introduced into the analyzer. The concentration range for the most efficient operation of the EAA is from 1 to 1,000 mg/m^3 . Since stack gas concentrations usually exceed 1,000 mg/m^3 , sample dilution with clean air is usually required.

The analyzer first places a unipolar charge on the particles being measured. These charged particles are then passed through an electrostatic field imposed between the sample-cell wall and a central sampling aperture. For the imposed field level, a certain segment of the particles will have enough mobility to make it to the sampling aperture and be collected. These separated particles are counted with an optical particle counter or a charge counter. The strength of the imposed field is then increased in steps, each step allowing for a larger segment of particles to be separated and counted. The resulting data are numerically analyzed to determine the number size distribution. No information on the chemical composition of the particles is obtainable since the particles are not collected.



Cascade Impactors
use the inertia of
particles to
separate them into
size categories.

Provide a complete
particle size
classification of gas
streams.

Cascade Impactors

Cascade impactors are used most frequently to determine the particle size distribution of exhaust streams from industrial sources. Cascade impactors utilize the inertia of the particles to separate the particulate matter in the sample gas stream into a number of size categories. Impactors measure the aerodynamic diameter of the particles.

The mechanism by which an impactor operates is illustrated in Figure 3-9. This impactor is constructed using a succession of stages, each containing orifice openings with an impaction slide or collection plate opposite the openings. In each

stage, the gas stream passes through the orifice opening and forms a jet that is directed toward the impaction plate.

Particles will impact on the plate if their inertia is large enough to overcome the drag of the air stream as it moves around the plate. Since each successive orifice opening is smaller than those on the preceding stage, the velocity of the air stream, and therefore that of the dispersed particles, is increased as the gas stream advances through the impactor. Consequently, smaller particles eventually acquire enough momentum to break away from the gas streamlines to impact on a plate. Particles passing the last stage are collected on a filter. A complete particle size classification of the gas stream is therefore achieved.

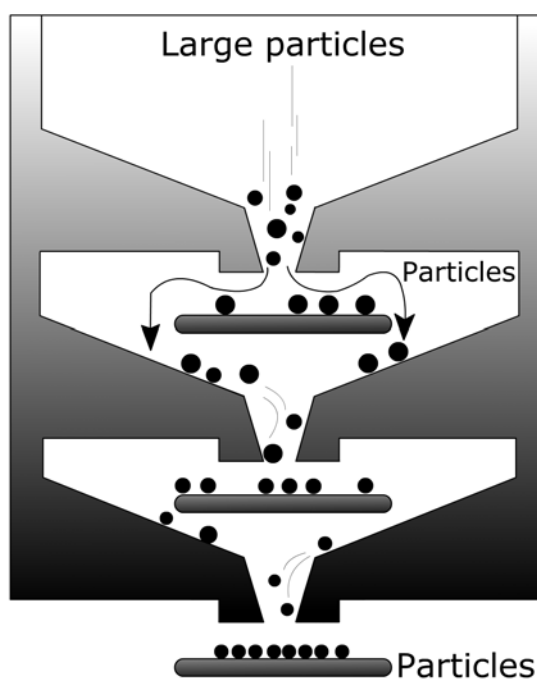


Figure 3-9. Operation of a cascade impactor

Typical impactors consist of a series of stacked stages and collection surfaces. Depending on the calibration requirements, each stage contains from one to as many as 400 precisely drilled jet orifices, identical in diameter on each stage but decreasing in diameter in each succeeding stage (Figure 3-10). Particles are collected on preweighed individual stages, usually filters made of glass fiber or thin metal foil. Some dusts are difficult to collect and require grease on the collection surface for adequate particle capture. The sampling period is usually in the range of 5 minutes to 30 minutes depending on the concentration of particulate matter in the gas stream being tested. It is important to avoid excessively long sampling periods because this can lead to the re-entrainment of particles initially captured on each stage. This results in a bias to lower-than-true measured particle sizes. Once the sampling is complete, the collection surfaces from each stage and the final filter are reweighed to determine the mass of particle in each size range. Chemical analyses

can also be performed on the separated particles.

The effective range for measuring the aerodynamic diameter is generally between 0.3 and 20 μm . However, some cascade manufacturers have achieved size fractionalization as small as 0.02 μm with low-pressure operation. Factors limiting the accuracy of cascade impactors include particle bounce on the impactor stages, particle re-entrainment from the impactor stages, and particle agglomerate fracturing in the impactor jets. The latter problem is caused by the high velocities created by the jets in subsequent stages. Other practical problems include the nucleation of vapors due to heat transfer to the large metal cascade impactor sampling heads and air infiltration into one or more of the numerous sealing surfaces of cascade impactor heads.

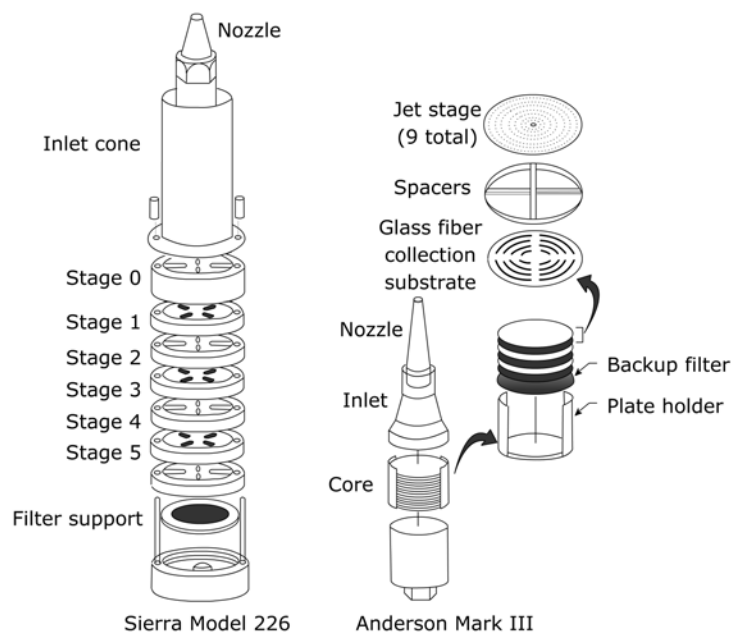


Figure 3-10. Cascade impactors



An ideal measuring instrument would measure size composition and real-time response.

Comparison of Particle Sizing Devices

Five particle-sizing instruments have been briefly described in the previous sections. Figure 3-11 shows the particle size range for each instrument, and Figure 3-12 compares their analytical capabilities. If an analyzer is able to resolve particle size or composition to the single particle level or to provide instantaneous real-time response, an open wedge symbol is indicated. A segmented wedge indicates that the analyzer is capable of providing particle size or composition information for discreet size ranges or for discreet time intervals. An integral symbol indicates that the particle sample is composited over some time period. As previously stated, the ideal measuring instrument would measure the exact size of each particle, determine the composition of each particle, and give an instantaneous real-time response. Accordingly, in Figure 3-12, open wedges are indicated in the size, time and composition columns for the ideal instrument.

3.3 Particle Size Distributions

Particulate matter emissions from both anthropogenic and biogenic sources do not consist of particles of just one size. Instead, they are composed of particles over a relatively wide size range. It is often desirable to describe this size range graphically or mathematically.

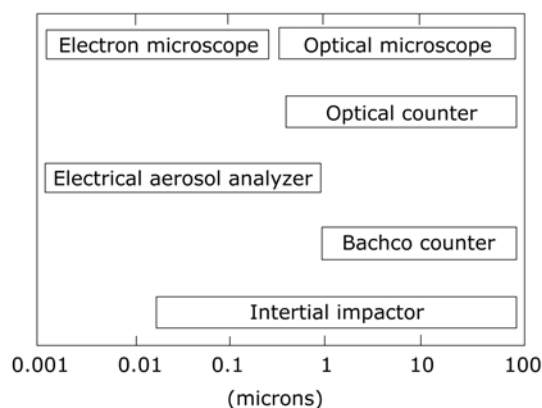


Figure 3-11. Size range capabilities of measuring devices

Figure 3-12. Comparison of particle sizing devices

Device	Size	Time	Composition
Ideal			
Microscope			
Optical counter			
EAA			
Bahco counter			
Impactor			

- Single particle level
- Discrete ranges
- Integrated averaging process

Figure 3-12. Comparison of particle sizing devices

One of the simplest means of describing a particle size distribution is a histogram as shown in Figure 3-13. This simply shows the number of particles in a set of arbitrary size ranges specified on the horizontal axis. The terms used to characterize

the particle size distribution are also shown in the figure.

The *median* particle size divides the frequency distribution in half: 50% of the mass has particles with a larger diameter, and 50% of the mass has particles with a smaller diameter. The *mean* is the mathematical average of the distribution and the mode is the particle size occurring most frequently. The value of the mean is sensitive to the quantities of particulate matter at the extreme lower and upper ends of the distribution.

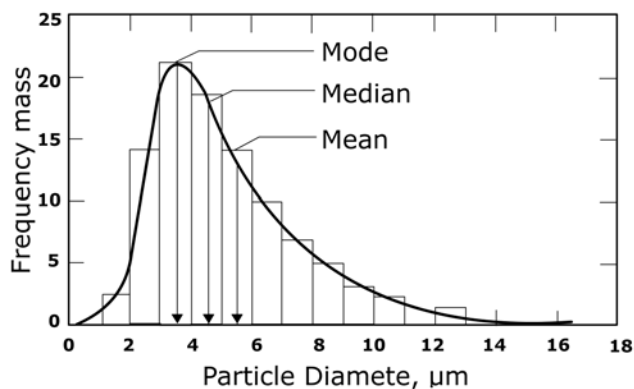


Figure 3-13. Particle size distribution

For many stationary and mobile sources, the observed particulate matter distribution in the effluent gas stream approximates what is known as a lognormal distribution. When the log of the particle diameter is plotted against the frequency of occurrence, a normal bell-shaped curve is generated. The histogram for a lognormal curve is shown in Figure 3-14. Note that the percent mass on the vertical axis is divided by the difference in the logarithms of the particle sizes defining the range ($\Delta \log d_p = \log d_{p,\max} - \log d_{p,\min}$). This is done to avoid any bias from the measurement range capabilities of the analyzer.

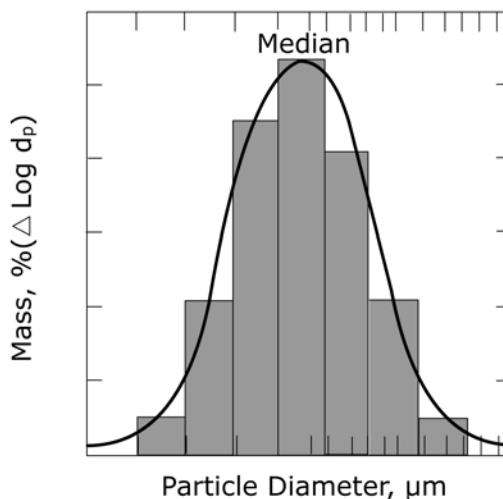


Figure 3-14. Lognormal size distribution

Lognormal distributions plot as a straight line on log-probability paper. This straight line can be characterized by two parameters: the intercept, represented by the geometric mass mean diameter, and the slope, represented by the geometric standard deviation. The geometric mass mean diameter is the particle size that is equivalent to the 50% probability point (zero standard deviation from the mean). The diversity of the particle sizes is described by the geometric standard deviation. A distribution with a broad range of sizes has a larger geometric standard deviation (σ_g) than one in which the particles are relatively similar in size. When the data are plotted in terms of the cumulative percent larger than size, the geometric standard deviation is determined by dividing the particle size at the 15.87 percent probability (-1 standard deviation from the mean) by the geometric mean size or by dividing the geometric mean size by the particle size at the 84.13 percent probability (+1 standard deviation from the mean):

(3-2)

$$\sigma_g = \frac{d_{15.87}}{d_{50}}$$

(3-3)

$$\sigma_g = \frac{d_{50}}{d_{84.13}}$$

Where

σ_g = geometric standard deviation of particle mass distribution

d_{50} = mass mean particle diameter

$d_{15.87}$ = particle diameter which 15.87% of the mass is larger than

$d_{84.13}$ = particle diameter which 84.13% of the mass is larger than

A plot of this type is illustrated in Example 3-1 below. If the data are plotted in terms of the cumulative percent smaller than size, the curve slopes the other way. Accordingly, the geometric standard deviation is given by the inverse of Equations 3-2 and 3-3. The easiest way to be sure of forming the proper ratio is to remember that d_{50} is always used in the ratio and that σ_g cannot be less than 1. A σ_g equal to 1 indicates that all of the particles are the same size. Thus, the ratio of d_{50} to either $d_{15.87}$ or $d_{84.13}$ must be such that it yields an answer that is greater than 1.

Particle size distributions resulting from complex particle formation mechanisms or several simultaneous formation mechanisms may not be lognormal. As shown in Figure 3-15, these distributions may exhibit more than one peak. In these cases,



plots of the data on log-probability paper will not yield a straight line.

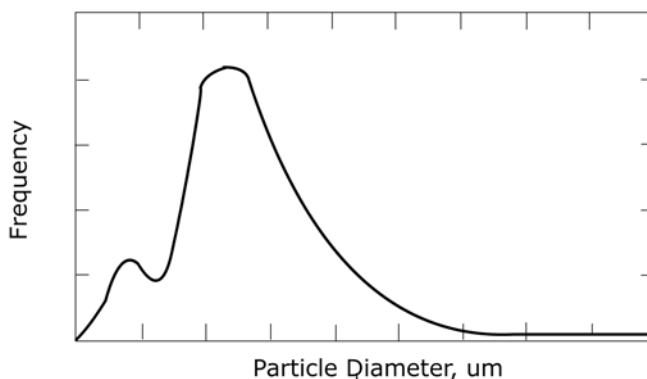


Figure 3-15. Bi-modal particle size distribution

Example 3-1 Determine the mass mean diameter and the geometric standard deviation of the particle collection represented by the following distribution:

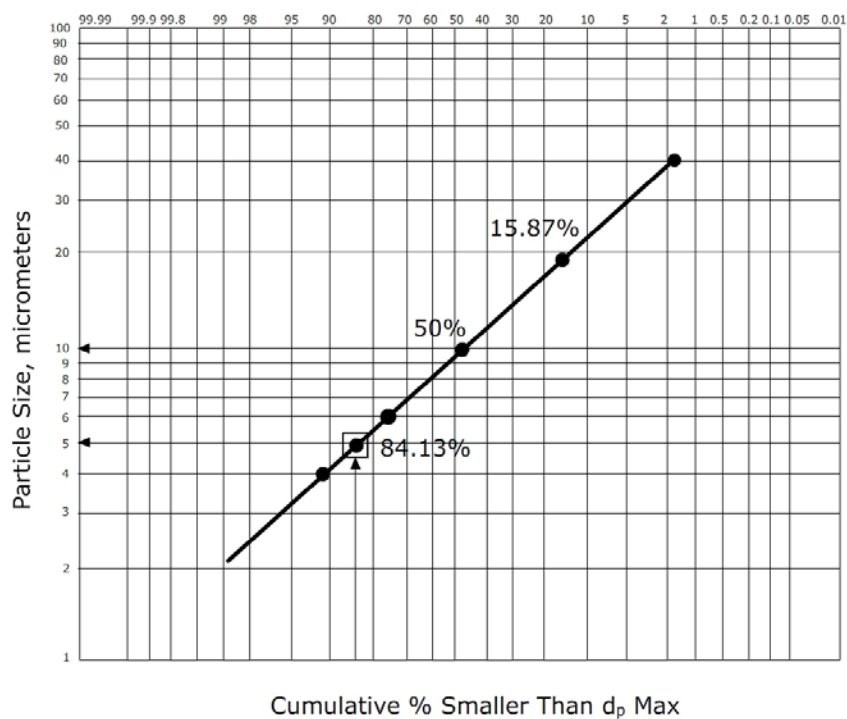
Size Range (gm)	Mass (mg)
<2	1.0
2 to 4	14.5
4 to 6	24.7
6 to 10	59.8
10 to 20	68.3
20 to 40	28.9
>40	2.8

Solution Refer to the following table. Determine the total mass and calculate the percentage in each size range. Starting with the size range for the smallest particles (<2 μm), subtract the percent mass in that range (0.50%) from 100.00 to determine the cumulative percent mass greater than 2 μm (99.50%). For each subsequent size range, subtract the percent mass in that range from the cumulative percent mass of the previous size range to determine the cumulative percent mass less than $d_p \text{ max}$ for that size range. For example, for the 2 to 4 μm size range, $99.50\% - 7.25\% = 92.25\%$, the cumulative percent mass greater than 4 μm .

Example Particle Size Data			
Size Range (μm)	Mass (mg)	Percent Mass in Size Range	Cumulative Percent Mass Less Than $d_p \text{ max}$
<2	1.0	0.50	99.50
2 to 4	14.5	7.25	92.25
4 to 6	24.7	12.35	79.90
6 to 10	59.8	29.90	50.00
10 to 20	68.3	34.15	15.85

20 to 40	28.9	14.45	1.40
>40	2.8	1.40	---
TOTAL	200.0	100.0	

Plot $d_{p,max}$ versus Cumulative Percent Mass Less Than $d_{p,max}$ on log-probability paper:



The mass mean particle diameter is found at the 50th percentile and is 10 μm . The geometric standard deviation is calculated from:

$$\sigma_g = \frac{d^{15.87}}{d^{50}} = \frac{20\mu\text{m}}{10\mu\text{m}} = 2.0$$

or

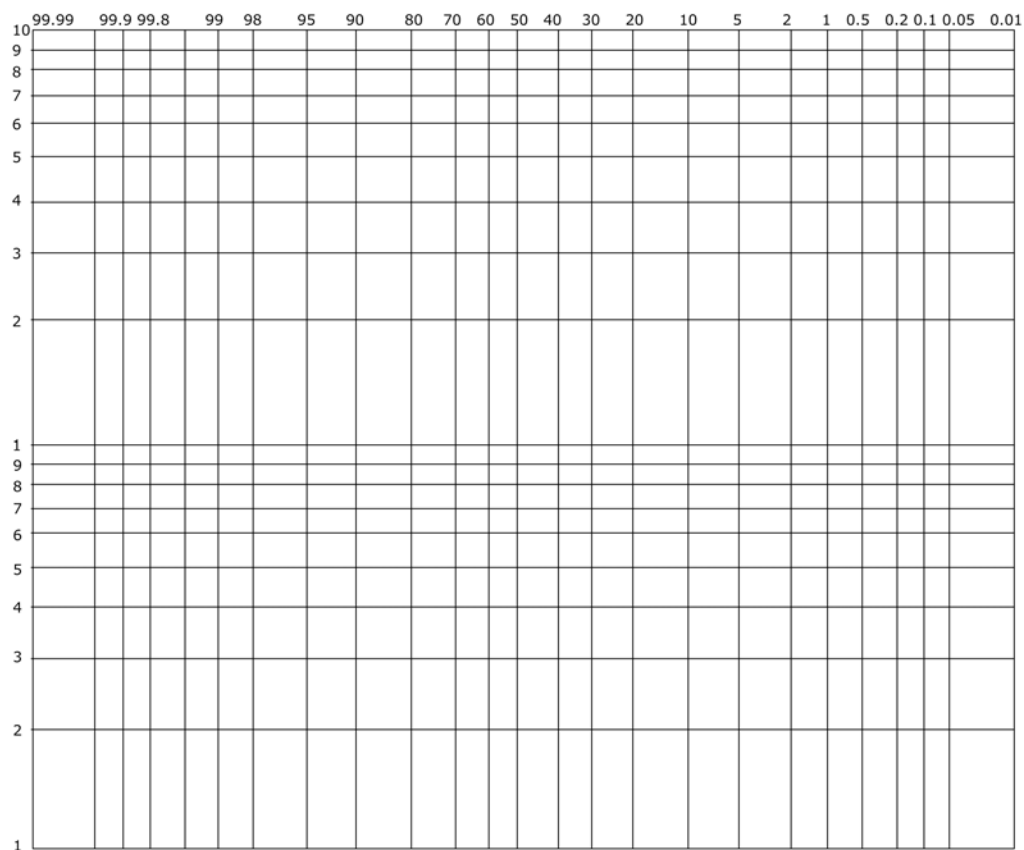
$$\sigma_g = \frac{d^{50}}{d^{84.13}} = \frac{10\mu\text{m}}{5\mu\text{m}} = 2.0$$



Review Problems

1. Calculate the aerodynamic diameter of a spherical particle having a true diameter of $2\text{ }\mu\text{m}$ and a density of 2.7 g/cm^3 .
2. Given the following distributions:
 - (a) Is either distribution lognormal?
 - (b) If yes, what is the geometric mass mean diameter and the geometric standard deviation?

Size Range (μm)	Sample A Mass (μg)	Sample B Mass (mg)
<0.6	25.50	8.50
0.6 to 1.0	33.15	11.05
1.0 to 1.2	17.85	7.65
1.2 to 3.0	102.00	40.80
3.0 to 8.0	63.75	15.30
8.0 to 10.0	5.10	1.69
>10.0	7.65	0.01



Review Problem Solutions

1. Calculate the aerodynamic diameter of a spherical particle having a true diameter of 2 μm and a density of 2.7 g/cm^3 .

Solution

Assume that the Cunningham slip correction factor is 1.

$$d_p = d \sqrt{\rho_p C_C} = 2 \sqrt{(2.7)(1.0)} = 3.29 \mu\text{m}$$

2. Given the following distributions:
 (a) Is either distribution lognormal?
 (b) If yes, what is the geometric mass mean diameter and the geometric standard deviation?

	Sample A	Sample B
Size Range (gm)	Mass (mg)	Mass (mg)
<0.6	25.50	8.50
0.6 to 1.0	33.15	11.05
1.0 to 1.2	17.85	7.65
1.2 to 3.0	102.00	40.80
3.0 to 8.0	63.75	15.30
8.0 to 10.0	5.10	1.69
>10.0	7.65	0.01

Solution for Sample A

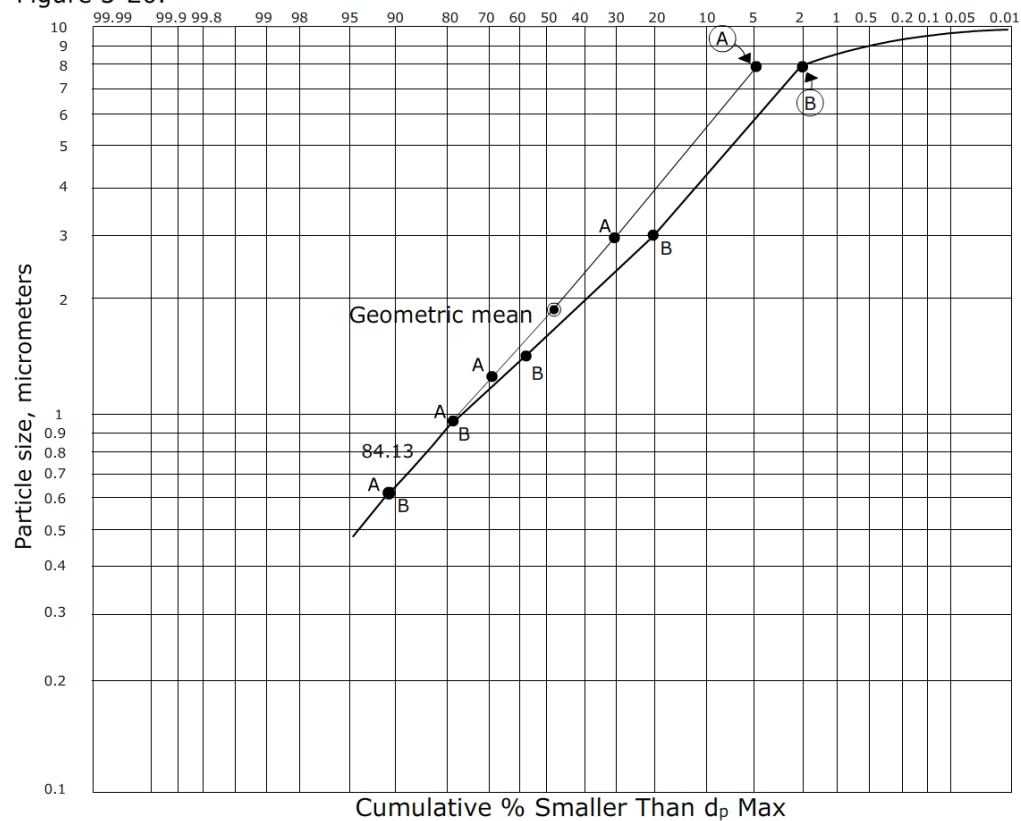
Size Range (gm)	Mass (mg)	Percent Mass in Size Range	Cumulative Percent Mass Less Than $d_{p\text{max}}$
<0.6	25.50	10	90
0.6 to 1.0	33.15	13	77
1.0 to 1.2	17.85	7	70
1.2 to 3.0	102.00	40	30
3.0 to 8.0	63.75	25	5
8.0 to 10.0	5.10	2	3
>10.0	7.65	3	---
TOTAL	255.0	100.0	

Solution for Sample B

Size Range (gm)	Mass (mg)	Percent Mass in Size Range	Cumulative Percent Mass Less Than d_{pmax}
<0.6	8.50	10	90
0.6 to 1.0	11.05	13	77
1.0 to 1.2	7.65	9	68
1.2 to 3.0	40.80	48	20
3.0 to 8.0	15.30	18	2
8.0 to 10.0	1.69	1.99	0.01
>10.0	0.01	0.01	---
TOTAL	85.0	100.0	

For each sample, plot d_{pmax} versus Cumulative Percent Mass Less Than d_{pmax} on log-probability paper:

Figure 3-20.



- Sample A is lognormal; Sample B is not lognormal.
- The geometric mass mean diameter and the geometric standard deviations for Sample A are:

$$d_{50} = 1.9 \mu\text{m} \quad \sigma_g = \frac{d_{50} = 1.9 \mu\text{m}}{d_{84.13} = 0.8 \mu\text{m}} = 2.4$$

References



Hewitt, G.W., *The Charging of Small Particles for Electrostatic Precipitation*, Paper No. 73283, Presented at the AIEE Winter General Meeting, New York, NY, 1957.

U.S. Environmental Protection Agency, *Guidelines for Particulate Sampling in Gaseous Effluents from Industrial Processes*, EPA 600/7-79-028, 1979.